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Can age-based restrictions replace horizontal lockdowns?

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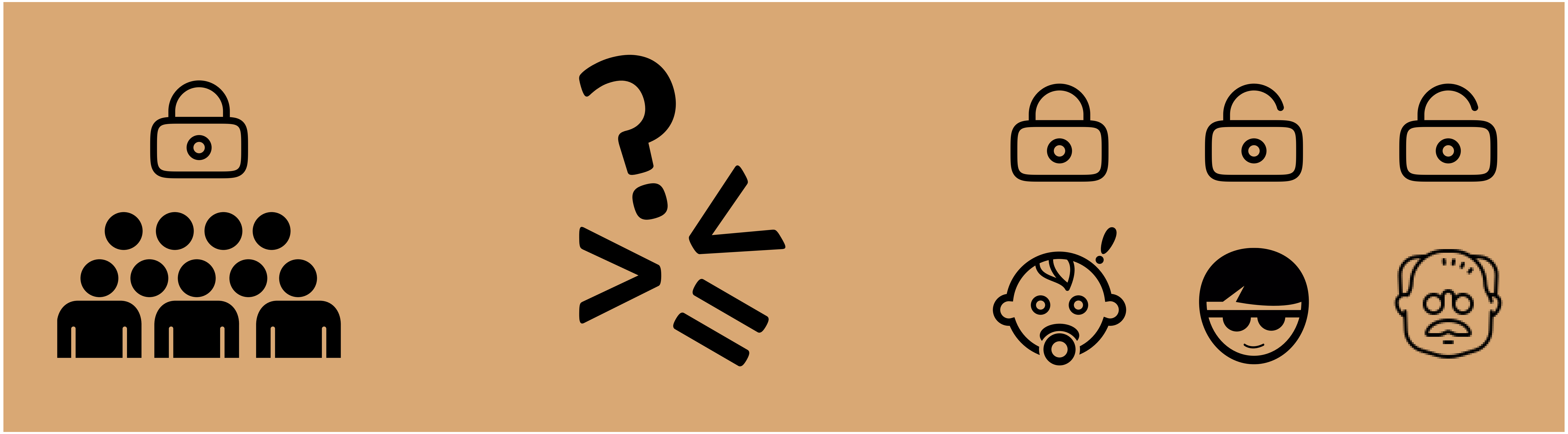
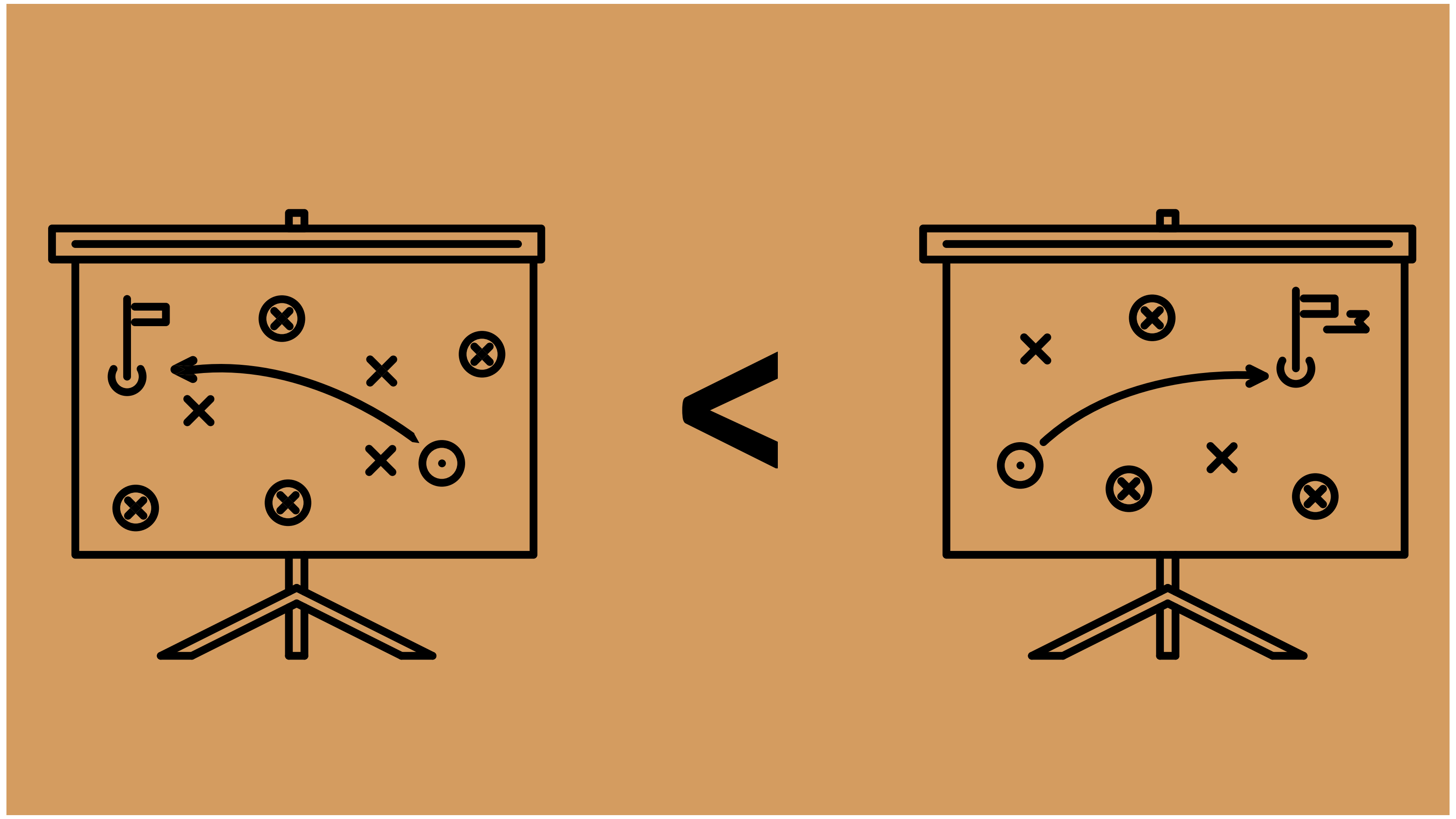
Joint work with **V. Bitsouni** (UPatras) & **N. Gialelis** (NKUA)

ECMTB'24

25 July, 2024



$$\frac{\partial S}{\partial \theta}$$



Agenda Overview

A Mathematical Model

- A1 Model description
- A2 Model analysis
- A3 Takeaways

B Comparison Framework

- B1 Background of the study
- B2 Definition of “strategy”
- B3 Description of the framework
- B4 Framework application
- B3 Takeaways

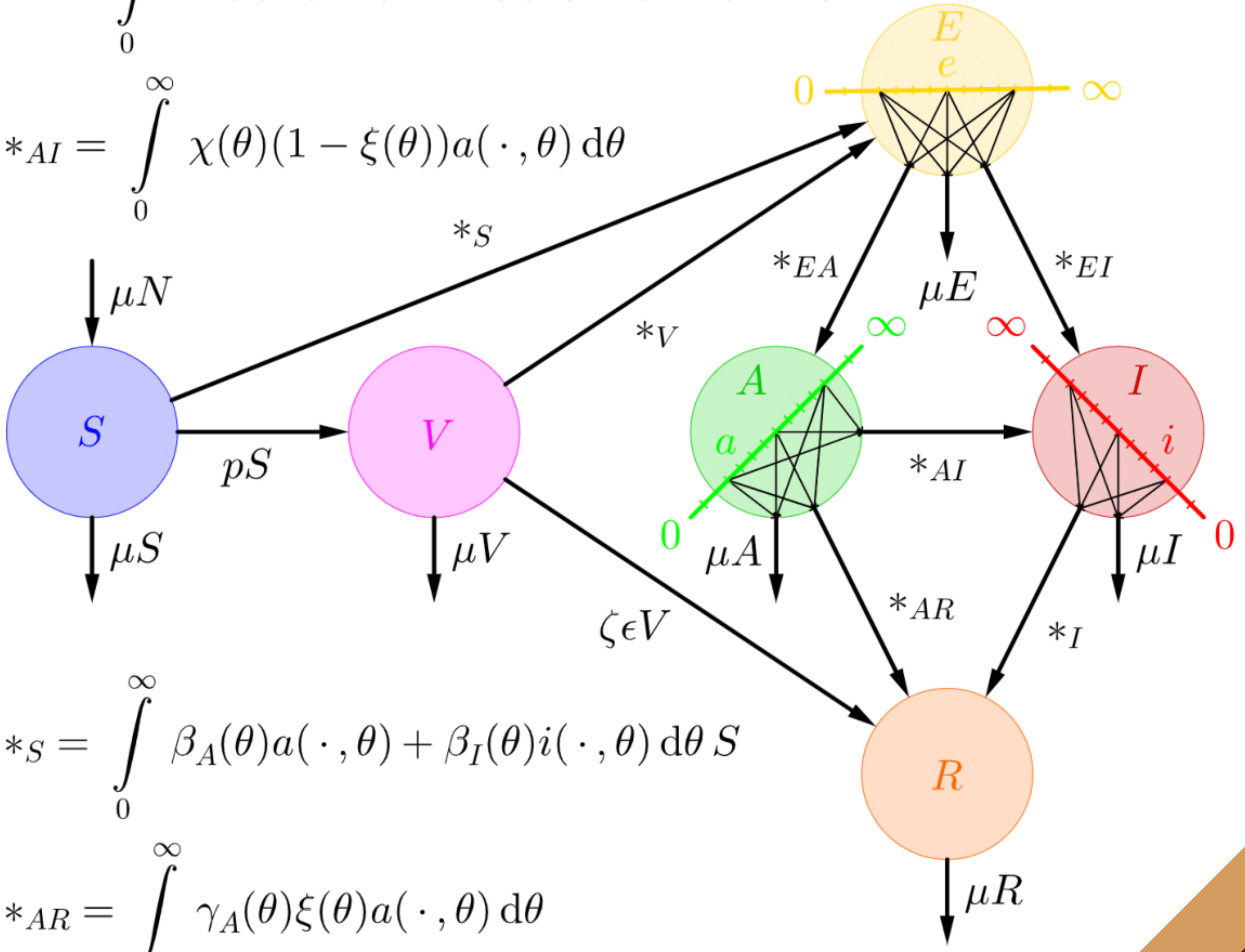
A Mathematical Model

A2 Model Description

$$*_{EA} = \int_0^\infty k(\theta)q(\theta)e(\,\cdot\,,\theta)\,\mathrm{d}\theta \qquad *_{EI} = \int_0^\infty k(\theta)(1-q(\theta))e(\,\cdot\,,\theta)\,\mathrm{d}\theta$$

$$*_V = \int_0^\infty \beta_A(\theta)a(\,\cdot\,,\theta) + \beta_I(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta\,(1-\epsilon)V$$

$$*_{AI} = \int_0^\infty \chi(\theta)(1-\xi(\theta))a(\,\cdot\,,\theta)\,\mathrm{d}\theta$$



$$*_S = \int_0^\infty \beta_A(\theta)a(\,\cdot\,,\theta) + \beta_I(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta\,S$$

$$*_{AR} = \int_0^\infty \gamma_A(\theta)\xi(\theta)a(\,\cdot\,,\theta)\,\mathrm{d}\theta$$

$$*_I = \int_0^\infty \gamma(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta$$

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}t} = \mu N_0 - \left(p + \int_0^\infty \beta_A(\theta)a(\,\cdot\,,\theta) + \beta_I(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta + \mu\right) S \\ S(0) = S_0, \end{cases}$$

$$\begin{cases} \dfrac{\mathrm{d}V}{\mathrm{d}t} = pS - \left(\zeta\epsilon + \int_0^\infty \beta_A(\theta)a(\,\cdot\,,\theta) + \beta_I(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta\,(1-\epsilon) + \mu\right) V \\ V(0) = V_0, \end{cases}$$

$$\begin{cases} \dfrac{\partial e}{\partial t} + \dfrac{\partial e}{\partial \theta} = -(k+\mu)\,e \\ e(\,\cdot\,,0) = \int_0^\infty \beta_A(\theta)a(\,\cdot\,,\theta) + \beta_I(\theta)i(\,\cdot\,,\theta)\,\mathrm{d}\theta\,(S + (1-\epsilon)V) \\ e(0,\,\cdot\,) = e_0, \end{cases}$$

$$\begin{cases} \dfrac{\partial a}{\partial t} + \dfrac{\partial a}{\partial \theta} = -(\gamma_A\xi + \chi(1-\xi) + \mu)\,a \\ a(\,\cdot\,,0) = \int_0^\infty k(\theta)q(\theta)e(\,\cdot\,,\theta)\,\mathrm{d}\theta \\ a(0,\,\cdot\,) = a_0, \end{cases}$$

$$\begin{cases} \dfrac{\partial i}{\partial t} + \dfrac{\partial i}{\partial \theta} = -(\gamma_I + \mu)\,i \\ i(\,\cdot\,,0) = \int_0^\infty k(\theta)\,(1-q(\theta))\,e(\,\cdot\,,\theta) + \chi(\theta)\,(1-\xi(\theta))\,a(\,\cdot\,,\theta)\,\mathrm{d}\theta \\ i(0,\,\cdot\,) = i_0. \end{cases}$$

A3 Model Analysis

For every $(S_0, V_0, e_0, a_0, i_0, R_0) \in (\mathbb{R}_0^+)^2 \times (L^1(\mathbb{R}_0^+;\mathbb{R}_0^+))^3 \times \mathbb{R}_0^+$

the problem is globally well-posed,

with $(S, V, e, a, i) \in (C^1(\mathbb{R}_0^+;\mathbb{R}_0^+))^2 \times (C(\mathbb{R}_0^+; L^1(\mathbb{R}_0^+)))^3$

Basic Reproduction Number

$$\mathcal{R}_0 := \frac{\mu N_0}{p+\mu}\left(1 + \frac{p(1-\epsilon)}{\zeta\epsilon+\mu}\right)(\mathcal{R}_A + \mathcal{R}_I) \ ,$$

$$\mathcal{R}_A := \int_0^\infty k(s)q(s)\mathrm{e}^{-\int_0^s k(\tau)+\mu \,\mathrm{d}\tau}\,\mathrm{d}s \int_0^\infty \beta_A(s)\mathrm{e}^{-\int_0^s \gamma_A(\tau)\xi(\tau)+\chi(\tau)(1-\xi(\tau))+\mu\,\mathrm{d}\tau}\,\mathrm{d}s$$

$$\begin{aligned}\mathcal{R}_I :=& \left(\int_0^\infty k(s)\,(1-q(s))\,\mathrm{e}^{-\int_0^s k(\tau)+\mu\,\mathrm{d}\tau}\,\mathrm{d}s\right.\\ &\left.+\int_0^\infty k(s)q(s)\mathrm{e}^{-\int_0^s k(\tau)+\mu\,\mathrm{d}\tau}\,\mathrm{d}s\int_0^\infty \chi(s)\,(1-\xi(s))\,\mathrm{e}^{-\int_0^s \gamma_A(\tau)\xi(\tau)+\chi(\tau)(1-\xi(\tau))+\mu\,\mathrm{d}\tau}\,\mathrm{d}s\right)\\ &\times\int_0^\infty \beta_I(s)\mathrm{e}^{-\int_0^s \gamma_I(\tau)+\mu\,\mathrm{d}\tau}\,\mathrm{d}s\end{aligned}$$

$$(S^*, V^*, e^*, a^*, i^*) = \left(\frac{\mu N_0}{p+\beta^*+\mu}, \frac{p\mu N_0}{(p+\beta^*+\mu)(\zeta\epsilon+\beta^*(1-\epsilon)\mu)}, e^*, a^*, i^*\right)$$

Equilibria Existence

If $\mathcal{R}_0 \leq 1$, then $(e^*, a^*, i^*) = (0, 0, 0)$

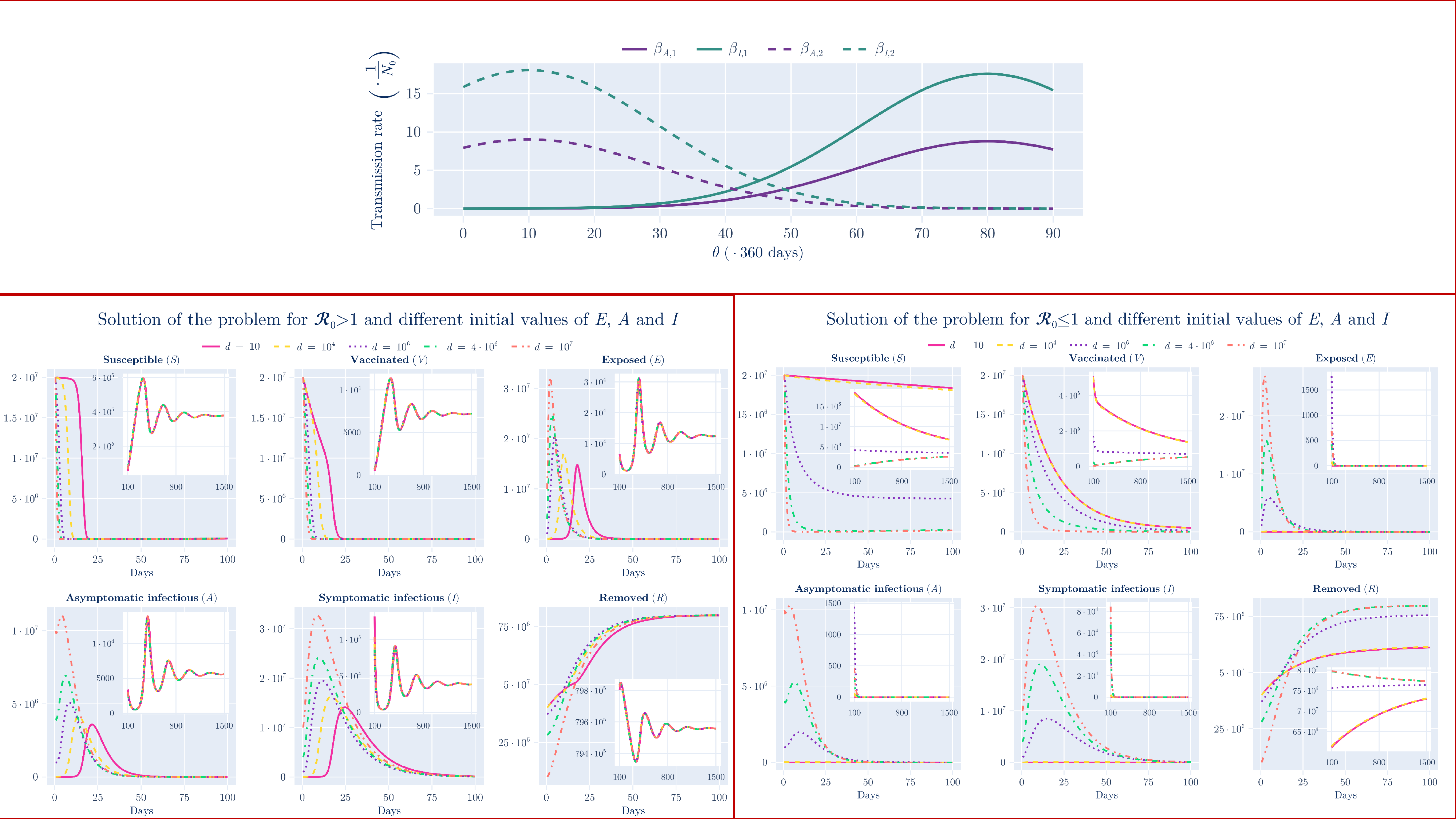
If $\mathcal{R}_0 > 1$, then

1. either $(e^*, a^*, i^*) = (0, 0, 0)$,
2. or $(e^*, a^*, i^*) > (0, 0, 0)$

Global Stability

If $\mathcal{R}_0 \leq 1$, then $(S^*, V^*, 0, 0, 0)$ is globally asymptotically stable

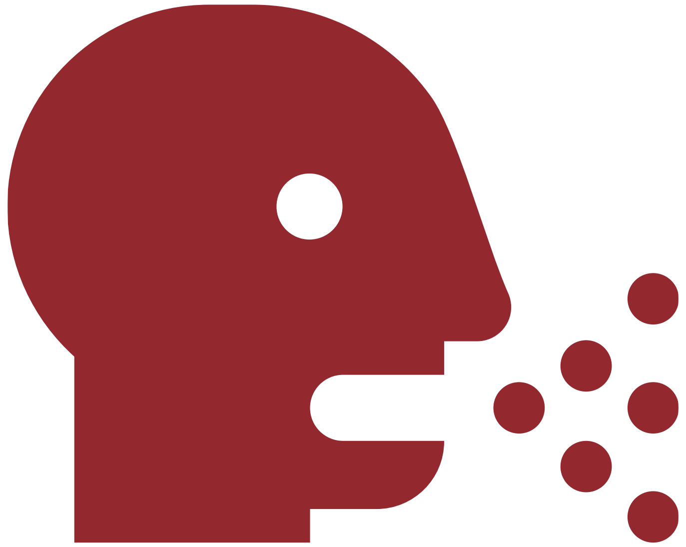
If $\mathcal{R}_0 > 1$, then $(S^*, V^*, e^*, a^*, i^*)$ with $(S^*, V^*, e^*, a^*, i^*) \neq (S^*, V^*, 0, 0, 0)$ is globally asymptotically stable



$\beta_{A,1}$ $\beta_{I,1}$ $\beta_{A,2}$ $\beta_{I,2}$
Transmission rate $\left(\cdot \frac{1}{N_0}\right)$
θ (· 360 days)
Solution of the problem for $\mathcal{R}_0 > 1$ and different initial values of E, A and I
$d = 10$ $d = 10^4$ $d = 10^6$ $d = 4 \cdot 10^6$ $d = 10^7$
Susceptible (S)
Vaccinated (V)
Exposed (E)
Asymptomatic infectious (A)
Symptomatic infectious (I)
Removed (R)
Days
Solution of the problem for $\mathcal{R}_0 \leq 1$ and different initial values of E, A and I
$d = 10$ $d = 10^4$ $d = 10^6$ $d = 4 \cdot 10^6$ $d = 10^7$
Susceptible (S)
Vaccinated (V)
Exposed (E)
Asymptomatic infectious (A)
Symptomatic infectious (I)
Removed (R)
Days

A4 Takeaways

The age-density of the transmission rates
can affect the outcome of the epidemic



The effects of the asymptomatic and symptomatic population
on the basic reproduction number
are different

V. Bitsouni, N. Gialelis, and V. Tsilidis,
An age-structured SVEAIR epidemiological model,
Mathematical Methods in the Applied Sciences (2024)

B Comparison Framework

B1

Background of the Study

After the COVID-19 pandemic
and the economical consequences of lockdowns,
finding alternative strategies
to intervene in disease spreading,
has been a hot topic in the scientific community

Relevance of the Study

Give a rigorous definition of the notion of epidemiological strategies

Scope of the Study

Propose a framework for systematically comparing certain epidemiological strategies

Research Question

Can age-based restrictions replace horizontal lockdowns, in the case of the SARS-CoV-2 pandemic?

B2

Definition of Strategy

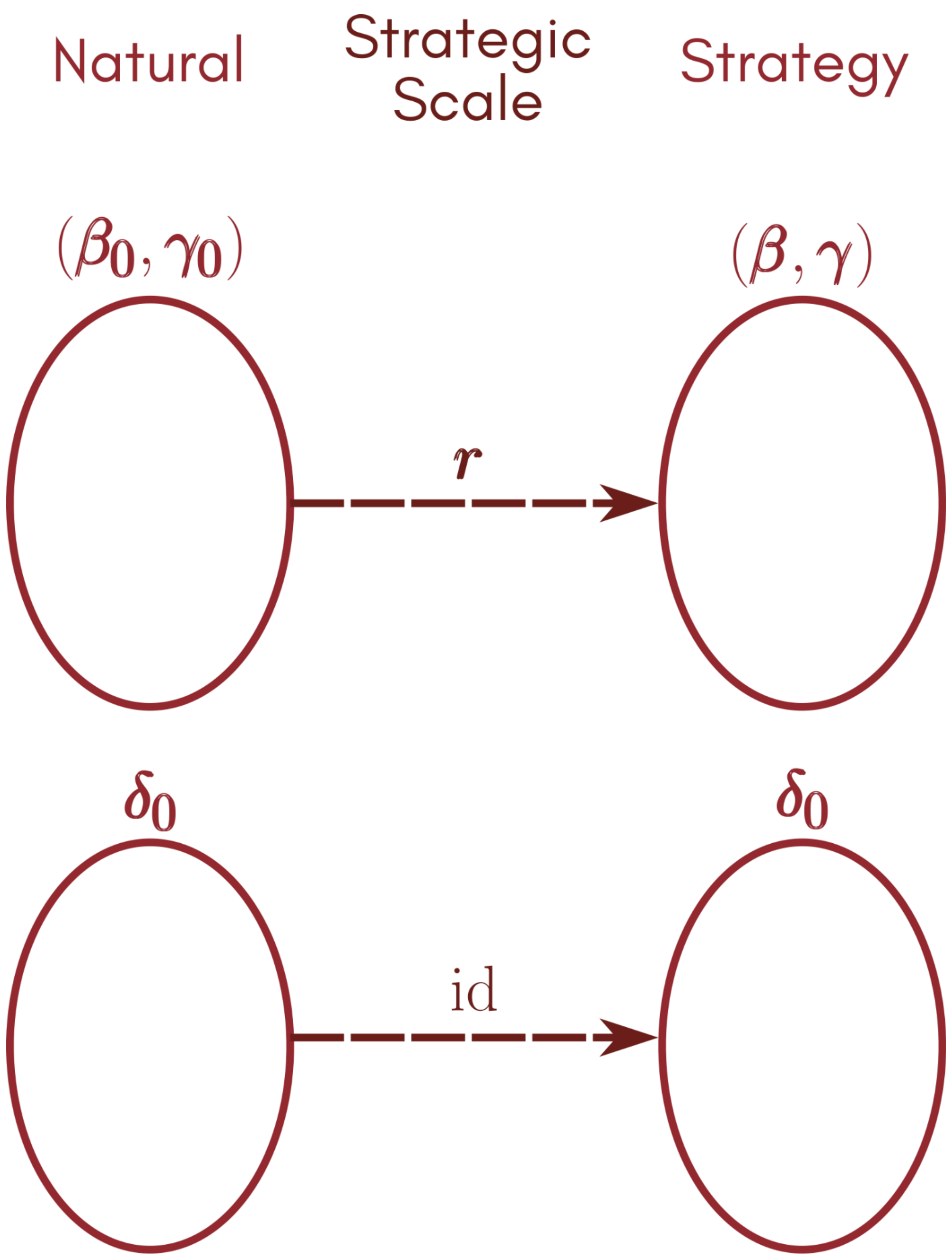
Strategy: Mathematical description
of a set of epidemiological interventions
made by potential external factors
in order to restrict the epidemiological phenomenon

Let $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \in F(\mathcal{X}; \mathcal{P}_{\mathrm{tr,r}})$.

A set $S = S(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0) \subseteq F(\mathcal{X}; \mathcal{P}_{\mathrm{tr,r}})$
is called a strategy with respect to $(\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$ iff

$$\forall \boldsymbol{y} \in S \; \exists \boldsymbol{r} = \boldsymbol{r}(\,\cdot\,; (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0), \boldsymbol{y}) \in F\left(\mathcal{X}; \mathbb{R}^{n_1+n_2}\right) \text{ s.t. } \boldsymbol{y} = \boldsymbol{r} \odot (\boldsymbol{\beta}_0, \boldsymbol{\gamma}_0)$$

The function $\boldsymbol{r}$ is called strategic scale of $\boldsymbol{y}$



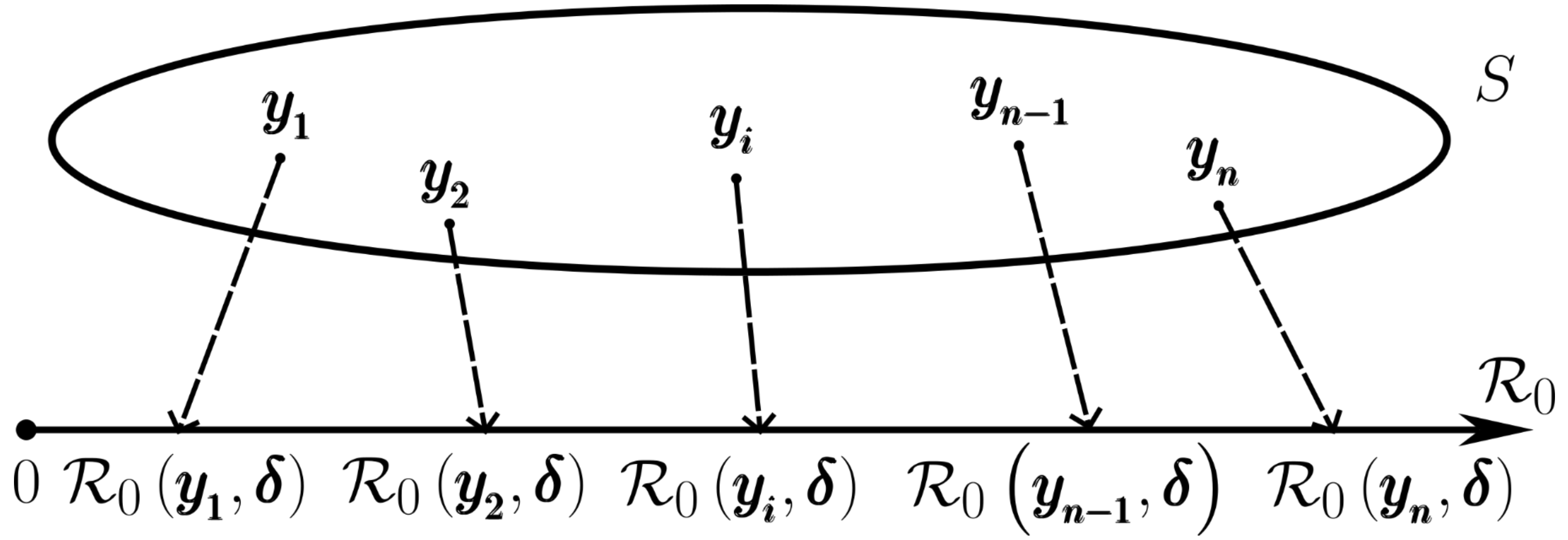
Comparison of Strategies

$\mathcal{R}_0$: a measure of comparison

$\mathcal{R}_0$ can be considered as a function of the parameters of a model

$$((\boldsymbol{\beta},\boldsymbol{\gamma}),\boldsymbol{\delta}) \mapsto \mathcal{R}_0((\boldsymbol{\beta},\boldsymbol{\gamma}),\boldsymbol{\delta})$$

A strategy S is gradable iff $\forall \boldsymbol{\delta}$ the function $\mathcal{R}_0|_S((\boldsymbol{\beta},\boldsymbol{\gamma}),\boldsymbol{\delta})$ is injective.



Let S be a gradable strategy and $\{\boldsymbol{y}_i\}_{i=1}^k \subseteq S$ be a family of pairwise distinct elements of S, such that

$$\underbrace{\mathcal{R}_0(\boldsymbol{y}_1,\boldsymbol{\delta})}_{=:G_1} < \cdots < \underbrace{\mathcal{R}_0(\boldsymbol{y}_k,\boldsymbol{\delta})}_{=:G_k}.$$

We call the pair $\left(S, \boldsymbol{G} = (G_i)_{i=1}^k\right)$ a graded strategy, while $\boldsymbol{G}$ is called a gradation of S and each of the $G_1,\ldots,G_k$ is called a grade of $\boldsymbol{G}$.

B3 Description of the Framework

First step: Put a gradation of a gradable strategy in the top row

S_1	G_1	G_2	$\cdots$	G_k

B3 Description of the Framework

Second step: Put a strategy, along with various substrategies, in the first column

S_2 \ S_1	G_1	G_2	$\cdots$	G_k
S_{2_1}				
S_{2_2}				
$\vdots$				
S_{2_ℓ}				

B3 Description of the Framework

Third step: **Compare each pair of substrategies of S_1 and S_2**

$S_2 \backslash S_1$	G_1	G_2	$\cdots$	G_k
S_{2_1}	$\star_{11}$	$\star_{12}$	$\cdots$	$\star_{1k}$
S_{2_2}	$\star_{21}$	$\star_{22}$	$\cdots$	$\star_{2k}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
S_{2_ℓ}	$\star_{\ell 1}$	$\star_{\ell 2}$	$\cdots$	$\star_{\ell k}$

$$\star_{ij} = \begin{cases} \checkmark & \text{if the } \mathcal{R}_0 \text{ of } S_{2i} \text{ is less than or equal to } G_j \\ ✗ & \text{otherwise ,} \end{cases} \quad \forall\,(i,j) \in \{1,\ldots,\ell\}\times\{1,\ldots,k\}$$

B3 Description of the Framework

Last step: **Calculate the percentage of checkmarks in each row and in each column**

S_2 \ S_1	G_1	G_2	$\cdots$	G_k	Epidemiological coverage $(\cdot 100\%)$
S_{2_1}	$\star_{11}$	$\star_{12}$	$\cdots$	$\star_{1k}$	$\frac{\#\{\star_{1j}=\checkmark\}_{j=1}^{k}}{k}$
S_{2_2}	$\star_{21}$	$\star_{22}$	$\cdots$	$\star_{2k}$	$\frac{\#\{\star_{2j}=\checkmark\}_{j=1}^{k}}{k}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
S_{2_ℓ}	$\star_{\ell 1}$	$\star_{\ell 2}$	$\cdots$	$\star_{\ell k}$	$\frac{\#\{\star_{\ell j}=\checkmark\}_{j=1}^{k}}{k}$
Social coverage $(\cdot 100\%)$	$\frac{\#\{\star_{i1}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\frac{\#\{\star_{i2}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\cdots$	$\frac{\#\{\star_{ik}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\frac{\#\{\star_{ij}=\checkmark\}_{i,j=1}^{\ell,k}}{\ell\cdot k}$

B3 Description of the Framework

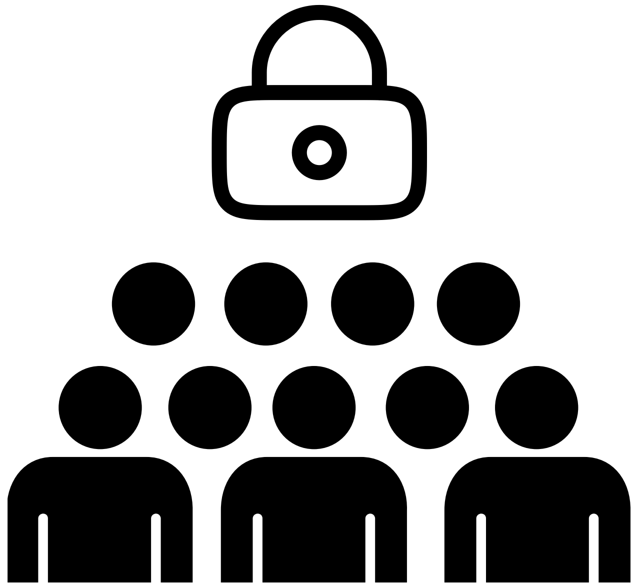
Last step: **Calculate the percentage of checkmarks in each row and in each column**

Already established strategy: S_1

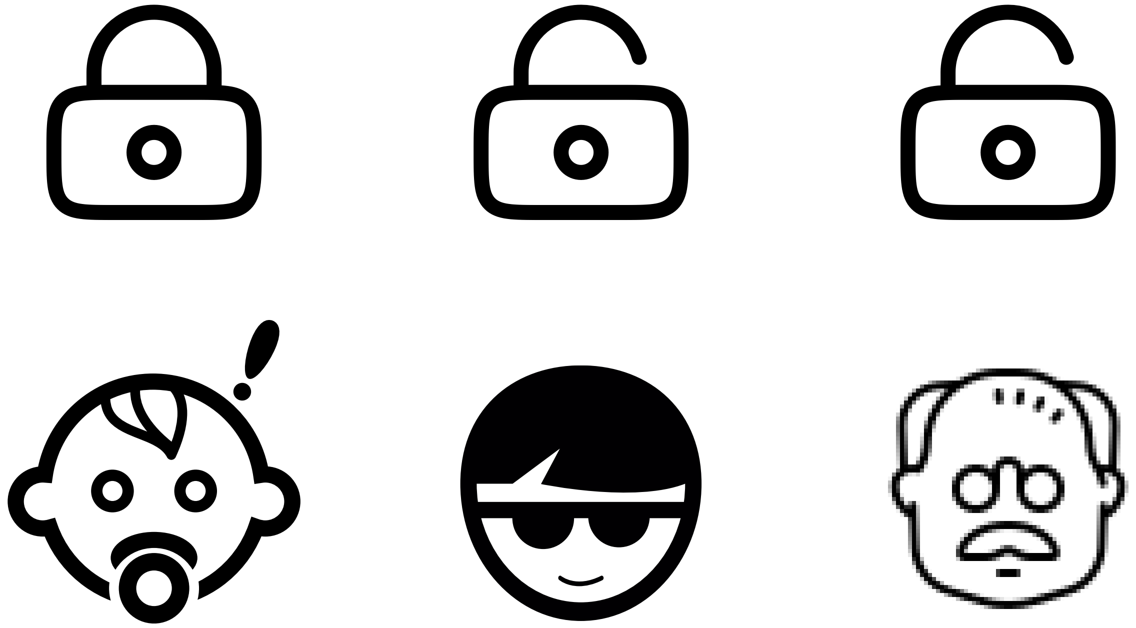
Potential alternative strategy: S_2

S_2 \ S_1	G_1	G_2	$\cdots$	G_k	Epidemiological coverage $(\cdot 100\%)$
S_{2_1}	$\star_{11}$	$\star_{12}$	$\cdots$	$\star_{1k}$	$\frac{\#\{\star_{1j}=\checkmark\}_{j=1}^{k}}{k}$
S_{2_2}	$\star_{21}$	$\star_{22}$	$\cdots$	$\star_{2k}$	$\frac{\#\{\star_{2j}=\checkmark\}_{j=1}^{k}}{k}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
S_{2_ℓ}	$\star_{\ell 1}$	$\star_{\ell 2}$	$\cdots$	$\star_{\ell k}$	$\frac{\#\{\star_{\ell j}=\checkmark\}_{j=1}^{k}}{k}$
Social coverage $(\cdot 100\%)$	$\frac{\#\{\star_{i1}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\frac{\#\{\star_{i2}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\cdots$	$\frac{\#\{\star_{ik}=\checkmark\}_{i=1}^{\ell}}{\ell}$	$\frac{\#\{\star_{ij}=\checkmark\}_{i,j=1}^{\ell,k}}{\ell\cdot k}$

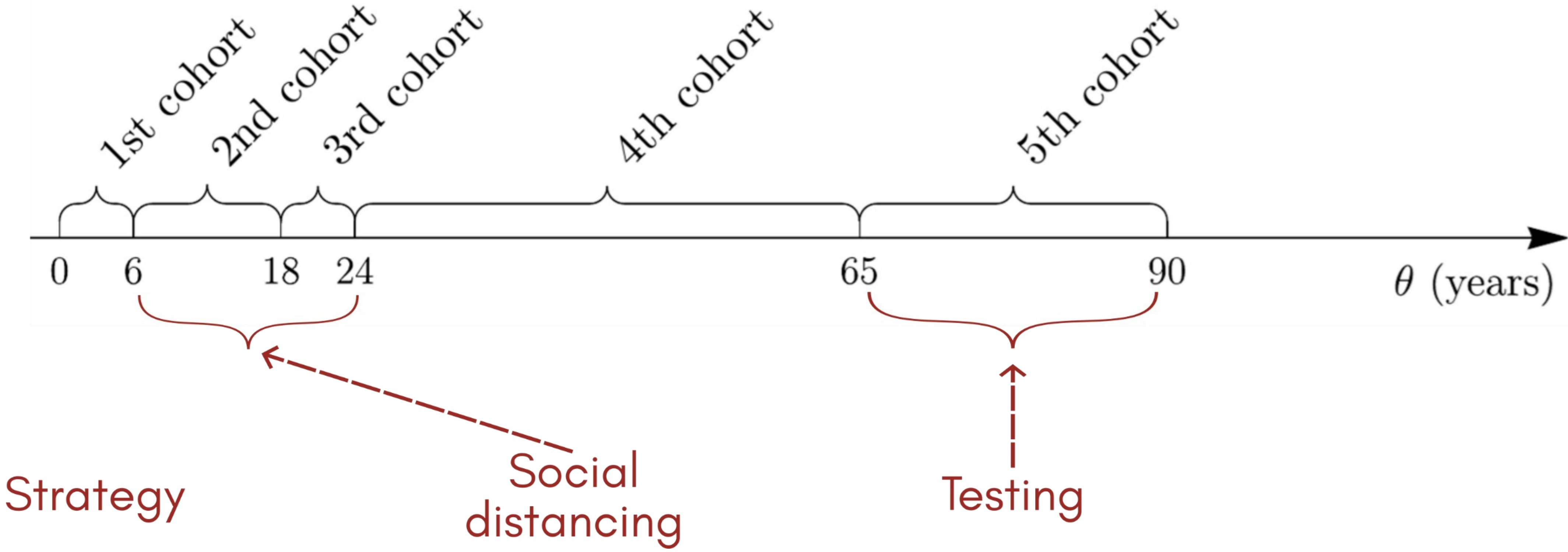
B4 Framework Application

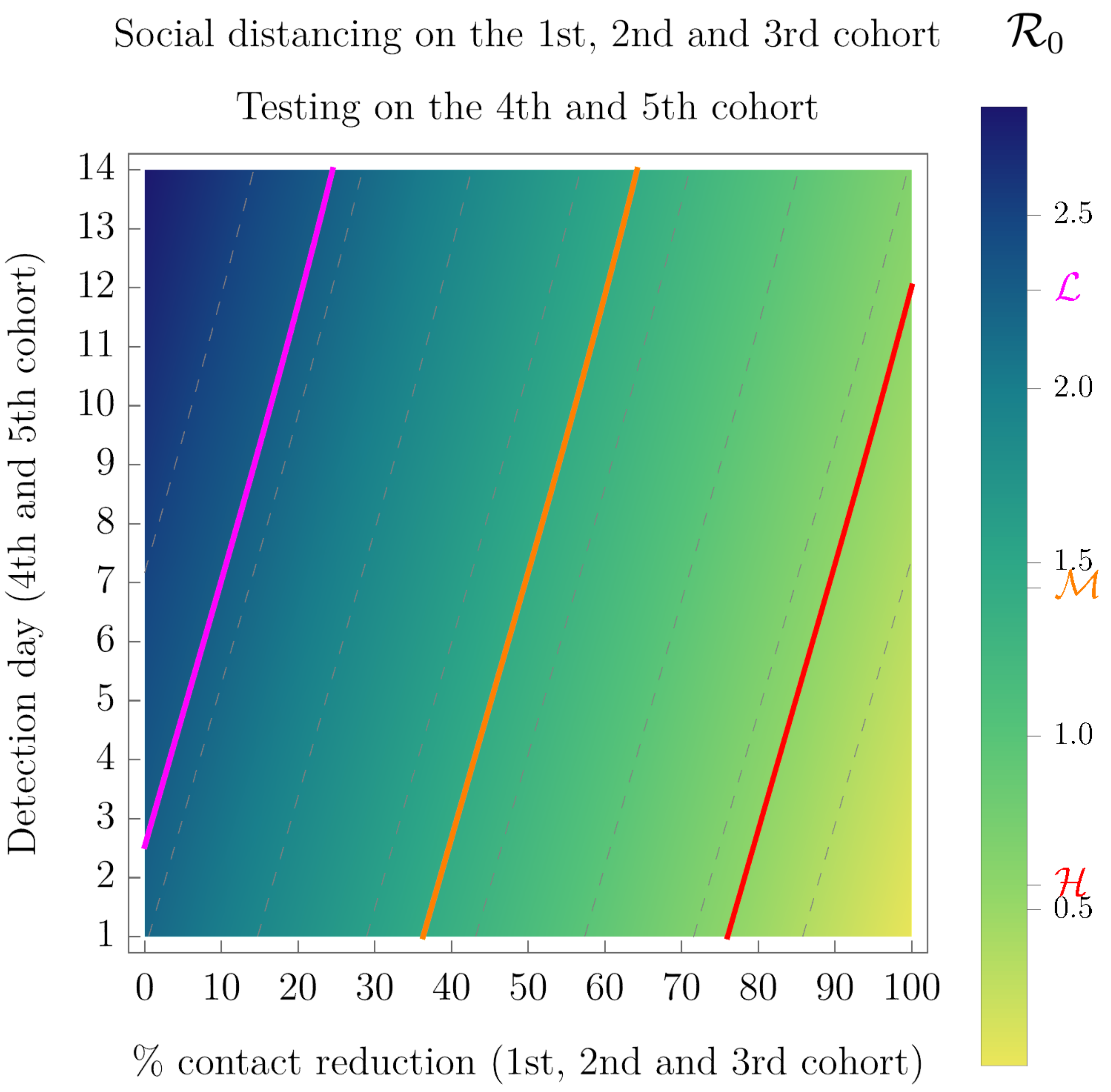


Gradation	Intensity level	Contact reduction	$\mathcal{R}_0$
G_1	High ($\mathcal{H}$)	80%	0.571
G_2	Medium ($\mathcal{M}$)	50%	1.427
G_3	Low ($\mathcal{L}$)	20%	2.283



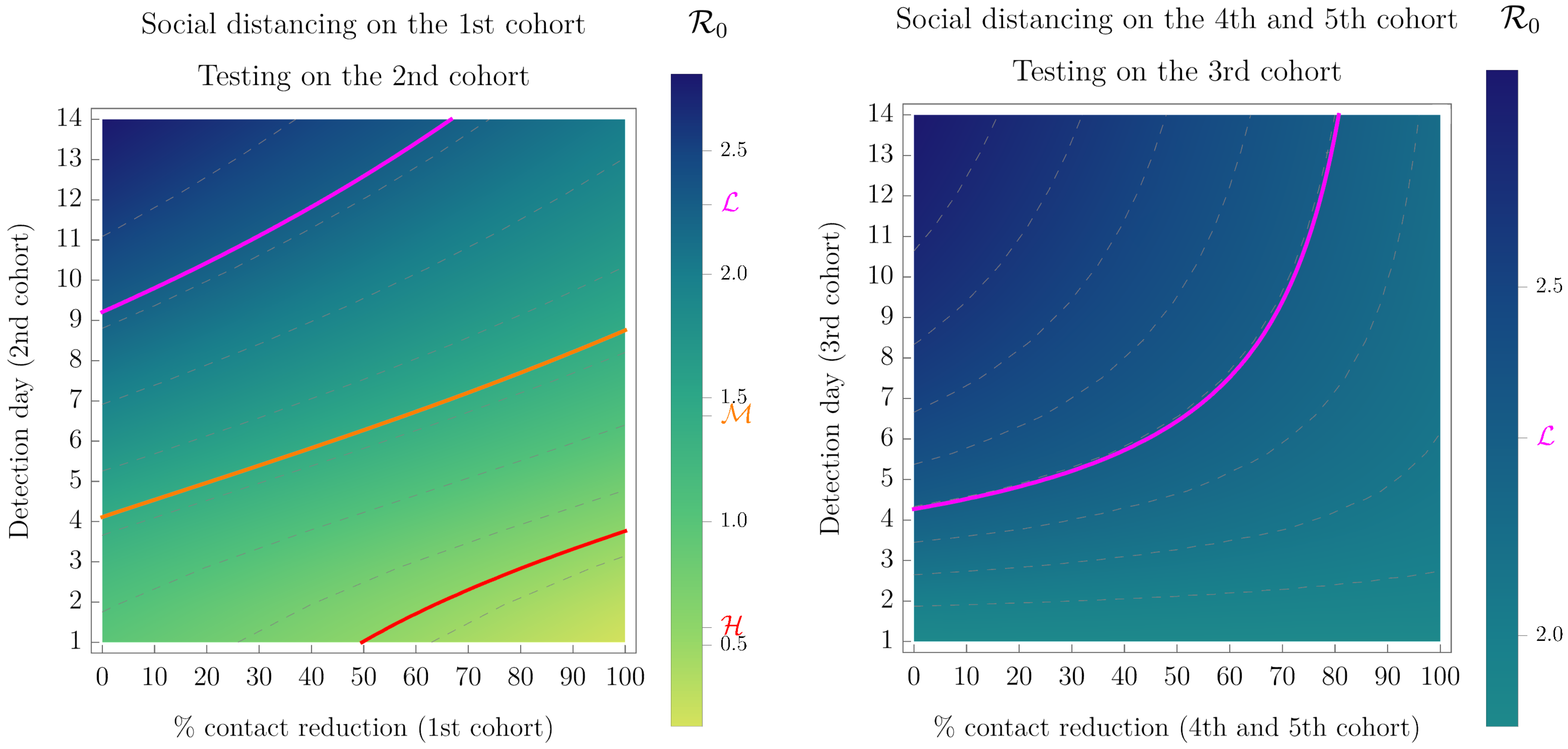
Distribution of popullation into cohorts





How long it takes
for the infectious individuals
of the selected cohorts
to be detected
through testing

Set of points
in which $\mathcal{R}_0$
is equal to the $\mathcal{R}_0$
of the medium intensity
lockdown strategy



Younger cohorts hold greater significance in reducing $\mathcal{R}_0$ than older cohorts

B5 Takeaways

A rigorous mathematical definition
of epidemiological strategies was proposed

A tool that allows policy-makers
to systematically compare
certain epidemiological strategies was created

Strategies that target the younger cohorts
have the best epidemiological coverage

V. Bitsouni, N. Gialelis, and V. Tsilidis,
A novel comparison framework for epidemiological strategies applied to age-based restrictions versus horizontal lockdowns,
Infectious Disease Modelling (2024)

Thank You!

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